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Nonlinear Model Predictive Control of Retrofit Heat-Exchanger Networks

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Abstract – Nonlinear Model Predictive Control (NMPC) of a Heat-Exchanger Network (HEN) is studied. For this purpose, an optimal retrofit design of the HEN is accomplished, and thus, bypasses as control variables are placed in order to satisfy steady-state hard/soft target-temperature constraints. The NMPC scheme, in which the nonlinear distributed-parameter HEN model is solved sequentially by referring to an algebraic steady-state-optimization model, satisfies the temporal and steady-state hard/soft constraints imposed on the target temperatures of the retrofit HEN.

Keywords: Heat-exchanger network, Optimal retrofit design, Nonlinear model predictive control

INTRODUCTION

A HEN must be flexible (resilient) to disturbances in its source streams to be able to control the network`s target-stream temperatures at specified values or within a permissible range. The degrees of freedom required to achieve control objectives can only be introduced by bypass and/or split streams. A method for the Optimal Retrofit Design (ORD) of HENs that do not posses control variables in their original structures has already been presented (Konukman et al., 1995; Boyacý et al., 1996; Uztürk et al., 1996; Uztürk and Akman, 1997). In the ORD method, bypasses, as control variables, are placed in order to satisfy hard/soft target-temperature constraints at the steady-state operating conditions, with minimum increase in exchanger areas (minimum retrofit-design cost). Given a disturbance, the optimal values of the target temperatures and bypass openings that guarantee steady-state feasibility of the new operating point can be computed by solving the ORD formulation. The remaining task is to determine how to apply the set of optimal bypass openings (controls) in time, such that the retrofit HEN’s dynamic response in reaching the final steady-state feasible operating point is acceptable/optimal.

A control scheme to be applied to a retrofit HEN should be able to: *a)* accommodate requirements of the ORD, *b)* match the optimal nominal operating conditions of the HEN exactly under zero-disturbance, *c)* satisfy all hard/soft target-temperature constraints for all possible disturbance directions at any post-disturbance steady-state feasible operating point, and *d)* satisfy all temporal hard/soft target-temperature constraints for all possible disturbance directions in transition from pre- to post-disturbance steady-state feasible operating points. For the control of retrofit HENs, the Nonlinear-Model-based Predictive-Control (NMPC) scheme should offer great flexibilities in satisfying the above-mentioned requirements. In this work, control of a retrofit HEN has been studied within the framework of NMPC.

OPTIMAL RETROFIT DESIGN OF HENs

Placement of bypass streams to a designed HEN in favor of its control creates a cost/controllability trade-off problem, the solution of which requires an ORD. Once the number and location of the bypass streams are decided, a HEN can be redesigned (fixed structure) in order to increase its static and dynamic flexibility against external disturbances. The objectives of the retrofit design can be set as follows. It should: *a)* be a minimum cost (total network area) design, *b)* yield the nominal operating conditions of the HEN exactly under zero-disturbance case, *c)* posses maximum flexibility in each disturbance direction, *d)* satisfy steady-state target constraints for all disturbance directions.

Formulation of the ORD problem is summarized by Eqs.1-5 (Uztürk and Akman, 1997). The decision variables are the bypass fractions (a different set for each disturbance direction) and the increments in exchanger areas (common for each disturbance direction). Disturbance (vertex) directions are identified by index *k*. The zero-disturbance direction (nominal operating conditions of the retrofit design) is identified by *k=0*. Equality constraints, Eq.2, guarantee that the target temperatures of the HEN under zero disturbance will exactly be those at the nominal operating condition. Inequality constraints, Eq.3, guarantee that the target-temperature range constraints are satisfied for all disturbance directions. Side constraints, Eq.5, put lower and upper limits on bypass openings only at zero-disturbance direction. Inequality constraints, Eq.4, limit the bypass openings, for each non-zero disturbance direction, between a certain fraction of openings of the corresponding bypasses at the nominal operating condition of the retrofit design (*k=0*).

$$\underset{S^k,\,\mathbf{u}^j,\,\mathbf{A}}{\textit{maximize}} \left\{ \sum_k S^k \Big/ \sum_n A_n \right\} \quad k=1,...,N_d \; j=0,...,N_d \qquad (1)$$

$$\mathbf{T}^{t,k} - \overline{\mathbf{T}}^{t} = 0 \qquad k=0 \qquad (2)$$

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$$\overline{\mathbf{T}}^t - \Delta\mathbf{T}^t \le \mathbf{T}^{t,k} \le \overline{\mathbf{T}}^t + \Delta\mathbf{T}^t \qquad \text{k=1,...,N}_\text{d} \qquad (3)$$

$$(1-r)\mathbf{u}^0 \le \mathbf{u}^j \le (1+r)\mathbf{u}^0 \qquad \text{j}\neq 0 \quad \text{k=1,...,N}_\text{d} \qquad (4)$$

$$u^{min} \le \mathbf{u}^j \le u^{max} \qquad \text{j=0} \qquad (5)$$

$\mathbf{u}^{min}$ and $\mathbf{u}^{max}$ for the nominal operating conditions of the retrofit design should be set by considering the realizability of the design. The upper limit should not be very high considering the bypass-line and valve sizing, yet it should not be very low considering the resiliency of the retrofit design ($\mathbf{u}^{min}$ should not be zero since, depending on the direction of a particular disturbance, some disturbances can be rejected or re-directed favorably only by decreasing bypass openings).

Static flexibility, S^k, (Eq.1) can be defined as the magnitude of the maximum disturbance (e.g., ± directed changes in source temperatures) that a structure can tolerate at steady state within desired operational limits (hard/soft target constraints). Determination of the flexibility of a structure is a nonlinear optimization problem, and for all disturbance directions, is directly obtained as the solution to the ORD formulation (Eqs.1-5).

STEADY-STATE OPTIMAL CONTROL OF HENs

For a HEN, the control objective is to keep the target temperatures either exactly at the nominal conditions for which the HEN has been designed (hard targets) or as close as possible to the nominal conditions (soft targets). The ORD formulation described above increases the region of feasible operation.

The steady-state optimal control (SSOC) problem can be defined as follows. When the HEN experiences disturbances in source temperatures, find a set of bypass fractions that will minimize the total deviations in the target temperatures, subject to steady-state target-temperature constraints (soft/hard) and manipulated-variable bounds, such that the HEN remains feasible. If the magnitude of the disturbance is less than or equal to the flexibility index, S^k, obtained from the solution of the ORD problem (Eqs.1-5) corresponding to this particular disturbance direction, then it is guaranteed that the SSOC problem will have a feasible solution. Formulation of the steady-state optimal control problem is summarized by Eqs.6-8 (Boyacý et al., 1996).

Once a feasible operating point is identified via SSOC, the remaining control task --assuming that there is no mismatch between the static model of the optimizer and the process at the steady state-- is to drive the system from its nominal operating conditions to the new feasible operating point (post-disturbance target) after a disturbance, i.e., the implementation of the optimal controls in time to reach $\mathbf{u}^{opt}(\infty)$.

$$\underset{\mathbf{u}}{minimize} \quad \left\| \Delta\mathbf{T}^t \right\| \qquad (6)$$

$$\Delta\mathbf{T}^t_{min} \le \Delta\mathbf{T}^t \le \Delta\mathbf{T}^t_{max} \qquad (7)$$

$$(1-r)\mathbf{u}^o \le \mathbf{u} \le (1+r)\mathbf{u}^o \qquad (8)$$

A SAMPLE HEN

The nominal HEN used in this work is shown in Fig.1. This nominal design can tolerate ± directed source-temperature disturbances with a magnitude of at most 5K (the minimum of the maximum flexibilities in each disturbance direction) when all the target temperatures are constrained within ±5K, i.e., no control is necessary if the objective is to keep the target temperatures within ±5K, as long as the disturbance entering the network from the "worst" disturbance direction is at most 5K in magnitude. However, among the 16 possible disturbance directions which do not include the zero disturbance in any of the source streams (four disturbance sources and two + and – directions), the nominal design, in its particular "best" disturbance direction, can tolerate source-temperature disturbances with magnitudes at most ±7.9K.

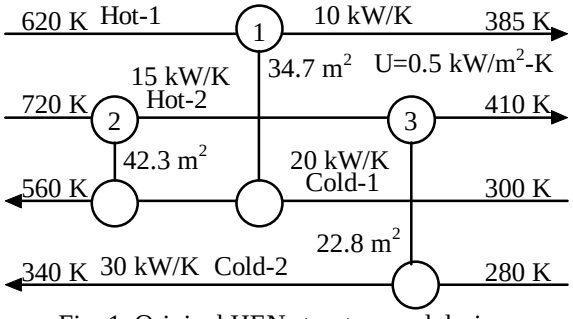


Fig. 1. Original HEN structure and design.

The control, and hence, the resiliency problems arise when the operational limits are tightened or when disturbances are expected in larger magnitudes. Once the number and location of the bypass streams are decided, a HEN can be redesigned without changing its original structure in order to increase its static and dynamic flexibility against external disturbances. The ORD formulation, Eqs.1-5, can be applied for each one of the bypass-location alternatives individually to obtain alternative retrofit HENs with greater steady-state and dynamic flexibilities based on steady-state model of the HEN. Fig.2 shows the final retrofit design of the HEN, location of bypasses, optimal nominal bypass openings, and pairings among controlled temperatures and manipulated bypasses. The locations of the bypasses are those that give maximum resiliency with relatively low retrofit cost (Uztürk and Akman, 1997). The static flexibility of the retrofit HEN is again 5K for the "worst" disturbance direction, however, in its "best" disturbance direction, the retrofit HEN can now tolerate disturbances with magnitudes at most ±12.4K.

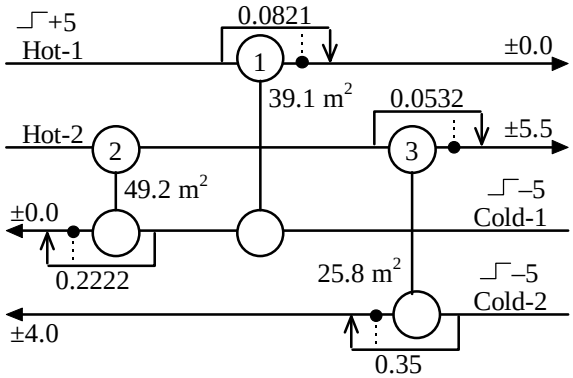


Fig. 2. The retrofit HEN.

DYNAMIC MODEL OF HEAT EXCHANGERS

For the dynamic modeling, single-tube-pass, shell-and-tube exchangers were chosen. The design parameters were evaluated using a design algorithm (Boyacý et al., 1996). The distributed-parameter models of the heat exchangers, given by Eqs.9-10, were used to construct the dynamic model of the HEN by connecting the PDEs of the individual exchangers through boundary conditions. Eqs.11-12 describe the energy balances at

the bypass-mixing points that are assumed algebraic (instantaneous mixing).

$$\frac{\partial T_H}{\partial t} = -v_H \frac{\partial T_H}{\partial z} + \frac{h_H a_H h_C a_C}{h_C a_C + h_H a_H} \frac{\left(T_C - T_H\right)}{A_H \rho_H C_{p_H}} \quad (9)$$

$$\frac{\partial T_C}{\partial t} = +v_C \frac{\partial T_C}{\partial z} + \frac{N_T h_C a_C h_H a_H}{h_C a_C + h_H a_H} \frac{\left(T_H - T_C\right)}{A_C \rho_C C_{p_C}} \quad (10)$$

bypass mixing cold side: $T_C^t = T_C^o u_C + T_C\left(1-u_C\right)$ (11)

bypass mixing hot side: $T_H^t = T_H^o u_H + T_H\left(1-u_H\right)$ (12)

Utilizing the "method of lines" numerical technique, the PDEs were discretized in z-direction using "five-point biased-upwind" finite-difference formulas (which eliminated the steady-state offset problem), and thus, converted into ODEs.

NMPC OF RETROFIT HENs

The objective in this section is to apply the NMPC technique (Bequette, 1991; Akman, 1997) to control of the retrofit HEN (Fig.2) by selecting a set of future control moves (bypass openings) in the control horizon to minimize the norm of the target-temperature deviations from predicted optimal/feasible post-disturbance steady-state values over a prediction horizon in which both the dynamic and steady-state models of the HEN are utilized subject to temporal and steady-state target constraints and manipulated-variable bounds. The NMPC scheme for control of retrofit HENs is depicted in Fig.3.

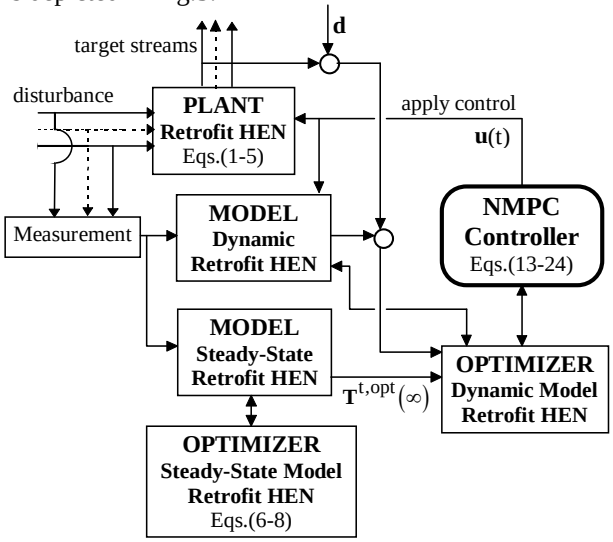


Fig. 3. The NMPC scheme for control of retrofit HENs.

The mathematical formulation of the general NMPC approach for control of HENs can be described by Eqs.13-24.

$$\underset{\mathbf{u}(i),\ldots,\mathbf{u}(i+m-1)}{minimize} \quad \Phi\left(\mathbf{u}(i)\right) = \int_{t_i}^{t_{k+R}} \left\| \mathbf{T}_k^{t,opt}(\infty) - \mathbf{T}_{MP}^t(i) \right\| dt$$

i=k,...,k+R (13)

$$\mathbf{T}_{MP}^t(i) = \mathbf{T}_M^t(i) + \mathbf{d}(k) \qquad \text{i=k,...,k+R} \quad (14)$$

$$\mathbf{d}(k) = \mathbf{T}_P^t(k) - \mathbf{T}_M^t(k) \quad (15)$$

$$\dot{\mathbf{x}}(t,z) = \boldsymbol{f}\left(\mathbf{x}(t,z), \partial\mathbf{x}(t,z)/\partial z, \mathbf{u}(t), \mathbf{p}, \mathbf{d}\right)$$

t ∈ [k,...,k+R] (16)

$$\mathbf{x}(k,z) = \bar{\mathbf{x}}(z) \quad (17)$$

$$\Delta\mathbf{T}_{min}^{t,*} \le \left\{\mathbf{T}^t(t) - \overline{\mathbf{T}}^t\right\} \le \Delta\mathbf{T}_{max}^{t,*} \quad \text{t ∈ [k,...,k+R]} \quad (18)$$

$$\left(1-r\right)\bar{\mathbf{u}} \le \mathbf{u}(i) \le \left(1+r\right)\bar{\mathbf{u}} \qquad \text{i=k,...,k+R} \quad (19)$$

$$-r^*\mathbf{u}(i) \le \Delta\mathbf{u}(i+1) \le r^*\mathbf{u}(i) \qquad \text{i=k+1,...,k+R} \quad (20)$$

$$\Delta\mathbf{u}(i+1) = \mathbf{u}(i+1) - \mathbf{u}(i) \quad (21)$$

$$\mathbf{u}(i) = \mathbf{u}(k+m-1) \qquad \forall\ \text{i > k+m−1} \quad (22)$$

$$\dot{x}(t) = \left\| min\left\{0, \left(\mathbf{T}^t(t) - \overline{\mathbf{T}}^t + \Delta\mathbf{T}_{min}^{t,*}\right)\right\}\right\| + \left\| min\left\{0, \left(\overline{\mathbf{T}}^t - \mathbf{T}^t(t) + \Delta\mathbf{T}_{max}^{t,*}\right)\right\}\right\|$$

t ∈ [k,...,k+R] (23)

$$x(t)\big|_{k+R} = 0 \qquad \text{t=k+R} \quad (24)$$

The objective function (Eq.13) is the norm of the residuals between the outputs (target temperatures) predicted by the HEN model and the set points (optimal/feasible post-disturbance steady-state target temperatures) over the prediction horizon of *R* time steps. The decision variables are the control actions considered only *m* time steps into the future (*m*<*R*). After the m^{th} time step control action is constant (Eq.22). The absolute bounds on the manipulated variables (Eq.19) are in accordance with those used in the formulation of the SSOC problem (Eq.8). The velocity constraints on the manipulated variables are defined by Eqs.20-21 where changes in the bypass openings are bounded within ±*r** fraction of the nominal bypass openings of the retrofit design. Eq.18 defines the temporal target-temperature constraints that should be in agreement with the target constraints used in the formulation of the SSOC problem (Eq.7). In the algorithm, only the first of the control actions, optimized over the prediction horizon, is implemented and the outputs (target temperatures) of the plant (actual HEN) are measured. Compensation of the plant/model mismatch is considered in Eqs.14-15 where the difference between the model output and the plant output at the current time step *k* is used to predict the output over the prediction horizon *R*. Eq.16 specifies the set of ODEs obtained after discretization of the PDEs (Eqs.9-10) describing the HEN dynamics. Eq.17 indicates the temperature profiles in the exchangers at the beginning of the k^{th} time step which are obtained from the solution of the model over the previous time step, based on the bypass openings actually implemented on the plant. The path constraints (Eq.18) are converted into a terminal equality constraint (Eq.24) by augmenting the set of ODEs with the single ODE given by Eq.23 (Li and Biegler, 1988).

NMPC OF EXAMPLE HEN WITH PLANT/MODEL MISMATCH

As an example, the control problem was defined as follows. When the consecutive source-temperature disturbances (0≤t<30: no disturbance, 30≤t<270: +5K in Hot-1, –5K in Cold-1 and –5K in Cold-2, 270≤t<510: no disturbance, 510≤t<750: +2K in Cold-1, 750≤t<1000: no disturbance) enter the HEN, keep the target temperatures of streams Hot-1 and Cold-1 at their nominal values given in Fig.1 (hard targets), keep the target temperature of stream Hot-2 within ±5.5K and Cold-2 within ±4K, and keep the bypass openings

within ±25% (r=0.25) of their nominal values indicated in Fig.2. The SSOC problem, Eqs.6-8, guarantees that the hard- and soft-target constraints are feasible for the retrofit HEN at the post-disturbance steady-state. The source of the plant/model mismatch was considered as a significant +2% underestimation of the convective heat-transfer coefficients in the PDE models, Eqs.9-10, of all the exchangers with respect to the plant. Table 1 lists some of the parameters used in the NMPC schemes.

Table 1. Some parameters used in the NMPC scheme.

$\Delta t = t_{k+1} - t_k = 30$ s	m = 3 steps R = 4 steps
$t_R - t_m = \Delta t$	r = 0.25 r* = 0.1

Fig.4 shows the response of the retrofit HEN and the controls implemented with the conventional NMPC scheme, with plant/model mismatch. The control scheme is capable of satisfying all the control objectives (steady-state and temporal hard/soft targets). However, the response is not smooth. The jumps in the target temperatures are due to stepwise implementation of the controls and the way the bypass-mixing points were modeled. These jumps cannot be avoided completely by imposing smaller velocity constraints on bypass openings. The response can be made smoother if the manipulated variables are parameterized as piecewise-continuous time-dependent functions in the prediction horizon (spline points $t_i,...,t_{i+m-1}$) and the first spline of each of the manipulated variables is applied in a sliding mode to the plant at t_k (Akman, 1997).

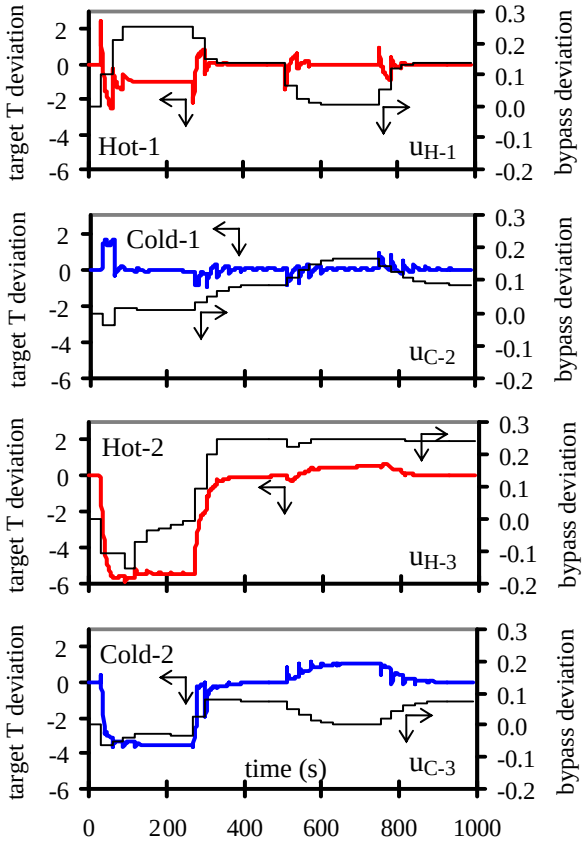


Fig. 4. Target temperature deviations from nominal values and deviations in bypass fractions relative to nominal values under conventional NMPC scheme, with plant/model mismatch.

CONCLUSIONS

Retrofit design of a HEN was performed considering placement of manipulated variables (bypasses) in order to increase HEN`s flexibility and resiliency. The NMPC of the retrofit HEN modeled with PDEs was studied. A steady-state optimization algorithm, which determines feasible (optimal) post-disturbance target temperatures, was combined with the NMPC algorithm. The conventional NMPC scheme satisfied all the temporal and steady-state target-temperature constraints even in the presence of a significant plant/model mismatch.

NOTATION

a	surface area of tube per unit length
A	heat-exchanger area or cross-sectional area
C_p	heat capacity
d	disturbance vector for plant/model mismatch
h	individual heat-transfer coefficient
k	current time step
m	number of future manipulated-variable moves
N_d	number of disturbance directions
N_T	number of tubes
p	vector of model parameters
r	fraction for steady-state bypass-opening bounds
r*	fraction for temporal bypass-opening bounds
R	prediction horizon
S	flexibility index (source-stream temperatures)
t	time
T	temperature
$\Delta T^{t,*}$	temporal constraints on target temperatures
u	vector of bypass fractions
U	overall heat-transfer coefficient
v	velocity
x	state variables
z	axial position
∞	steady-state value
ρ	density

Subscript		*Superscript*	
C	cold (shell) side	j, k	disturbance direction ID
H	hot (tube) side	o	zero-disturbance value
m	m^{th} bypass stream	opt	optimum
M	model	s	source
MP	model prediction	t	target
n	n^{th} heat exchanger	¯	nominal value
P	plant		

ACKNOWLEDGMENT

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Nonlinear Model Predictive Control of Retrofit Heat-Exchanger Networks

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ABSTRACT

- Control of a retrofit Heat-Exchanger Network (HEN) was studied within the framework of Nonlinear Model Predictive Control (NMPC).

- For this purpose, an optimal retrofit design of the HEN was performed, and thus, bypasses as control variables were placed in order to satisfy steady-state hard/soft target-temperature constraints.

- Control configuration was determined and tested under decentralized feedback-control scheme which did not satisfy the constraints.

- The NMPC scheme, in which the nonlinear distributed-parameter HEN model was solved sequentially, satisfied all the temporal and steady-state hard/soft target-temperature constraints.

- A version of the classical NMPC scheme, in-lined with piecewise-continuous control-vector parameterization, showed a superior performance with regard to smoothness of the response and significant reduction in computation time.

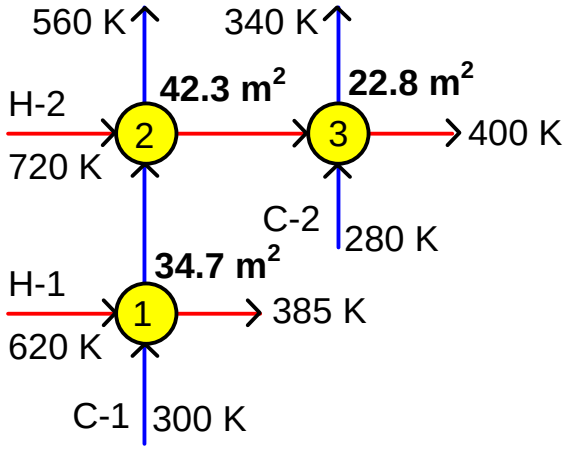
Keywords: *retrofit HEN, NMPC, NMPC with piecewise-continuous control-vector parameterization, input parameterization, basis function*

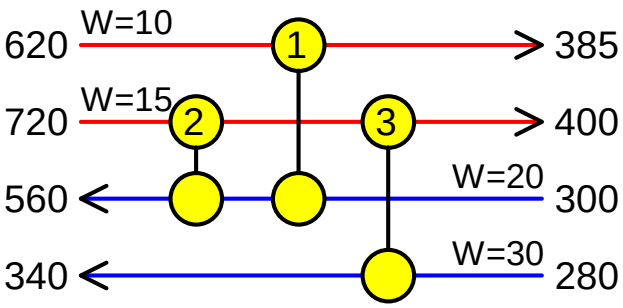
EXAMPLE HEN

Definition of Example HEN Problem (U_η = 0.5 kW/m^2-K)

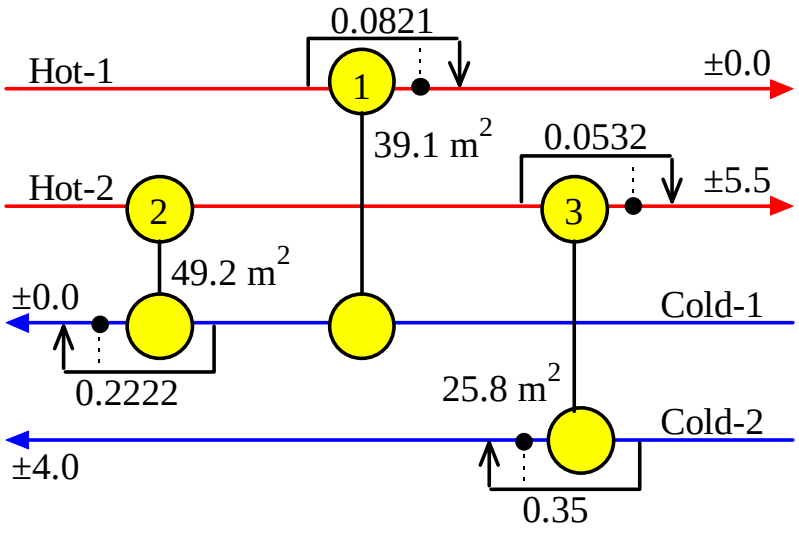
Stream	W (kW/K)	T_{in} (K)	T_{out} (K)
Hot-1	10	620	385
Hot-2	15	720	400
Cold-1	20	300	560
Cold-2	30	280	340

ORIGINAL HEN





RETROFIT HEN



DYNAMIC MODEL OF EXCHANGERS

- Single-tube-pass, shell-and-tube exchangers.
- Design parameters were evaluated using a detailed design algorithm.
- Distributed-parameter model of the heat exchangers were used to construct the dynamic model of the HENs by connecting the PDEs of the individual exchangers through boundary conditions.
- Utilizing the “method of lines” numerical technique, these PDEs were discretized in z-direction, and thus, converted into ODEs using “five-point biased-upwind” finite-difference formulas, which *eliminated steady-state offset* problem.

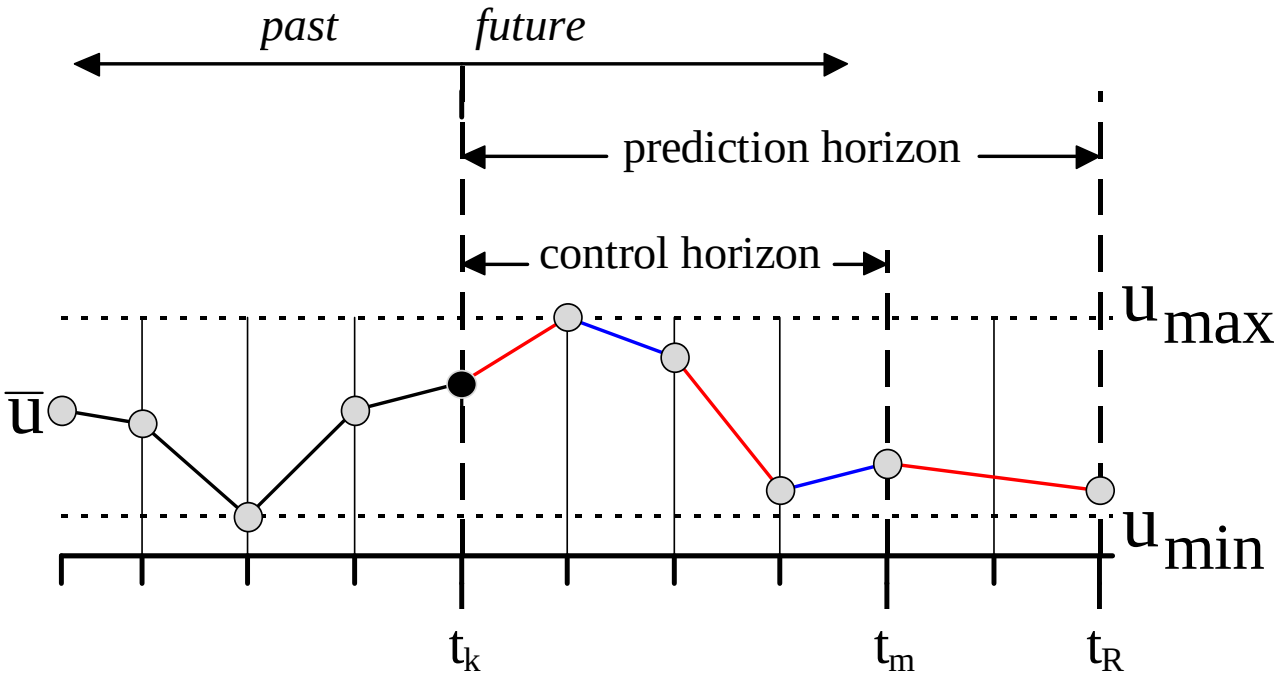
$$\frac{\partial T_H}{\partial t} = -v_H \frac{\partial T_H}{\partial z} + \frac{h_H a_H h_C a_C}{h_C a_C + h_H a_H} \frac{(T_C - T_H)}{A_H \rho_H C_{p_H}}$$

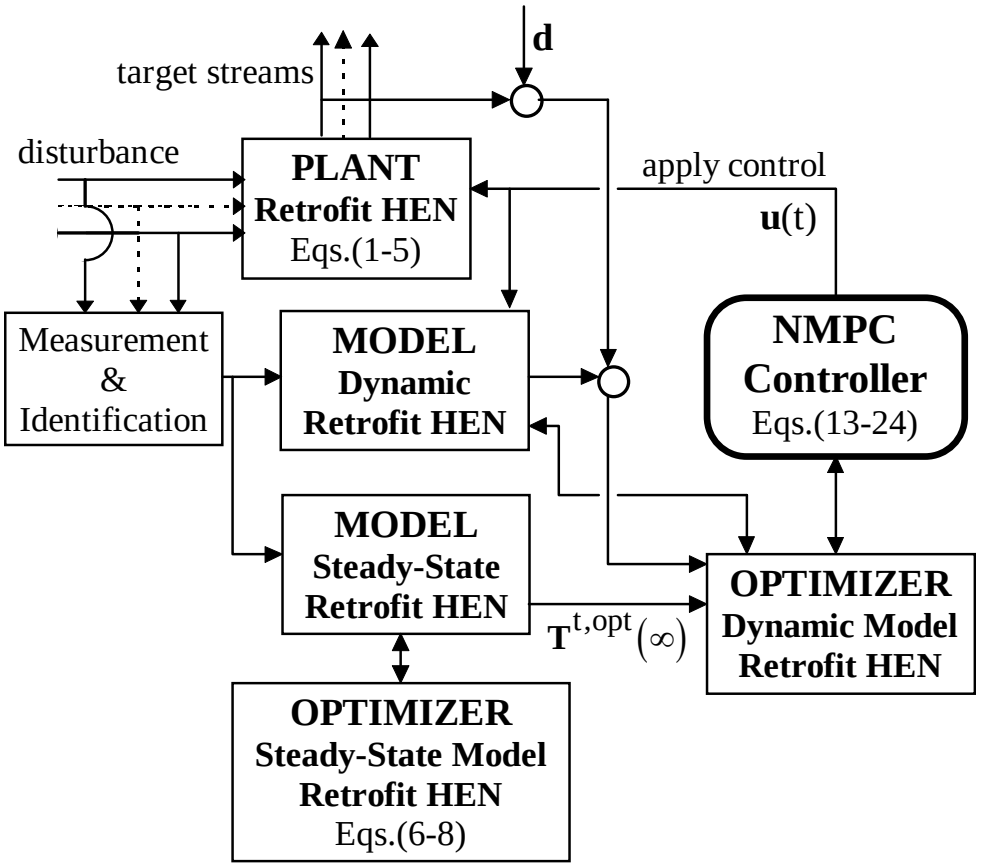
$$\frac{\partial T_C}{\partial t} = +v_C \frac{\partial T_C}{\partial z} + \frac{N_T h_C a_C h_H a_H}{h_C a_C + h_H a_H} \frac{(T_H - T_C)}{A_C \rho_C C_{p_C}}$$

bypass mixing cold side: $T_C^t = T_C^o u_C + T_C(1-u_C)$

bypass mixing hot side: $T_H^t = T_H^o u_H + T_H(1-u_H)$

NMPC OF RETROFIT HENs





FORMULATION OF NMPC FOR RETROFIT HENs

$$\underset{\mathbf{u}(i),\ldots,\mathbf{u}(i+m-1)}{\textit{minimize}} \quad \Phi\left(\mathbf{u}(i)\right) = \int_{t_i}^{t_{k+R}} \left\| \mathbf{T}_k^{t,opt}(\infty) - \mathbf{T}_{MP}^{t}(i) \right\| dt \qquad i=k,\ldots,k+R$$

$$\mathbf{T}_{MP}^{t}(i) = \mathbf{T}_{M}^{t}(i) + \mathbf{d}(k) \qquad i=k,\ldots,k+R$$

$$\mathbf{d}(k) = \mathbf{T}_{P}^{t}(k) - \mathbf{T}_{M}^{t}(k)$$

$$\dot{\mathbf{x}}(t,z) = \boldsymbol{f}\left(\mathbf{x}(t,z), \partial\mathbf{x}(t,z)/\partial z, \mathbf{u}(t), \mathbf{p}, \mathbf{d}\right) \qquad t \in [k,\ldots,k+R]$$

$$\mathbf{x}(k,z) = \bar{\mathbf{x}}(z)$$

$$\Delta\mathbf{T}_{min}^{t,*} \le \left\{ \mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} \right\} \le \Delta\mathbf{T}_{max}^{t,*} \qquad t \in [k,\ldots,k+R]$$

$$(1-r)\bar{\mathbf{u}} \le \mathbf{u}(i) \le (1+r)\bar{\mathbf{u}} \qquad i=k,\ldots,k+R$$

$$-r^{*}\mathbf{u}(i) \le \Delta\mathbf{u}(i+1) \le r^{*}\mathbf{u}(i) \qquad i=k+1,\ldots,k+R$$

$$\Delta\mathbf{u}(i+1) = \mathbf{u}(i+1) - \mathbf{u}(i)$$

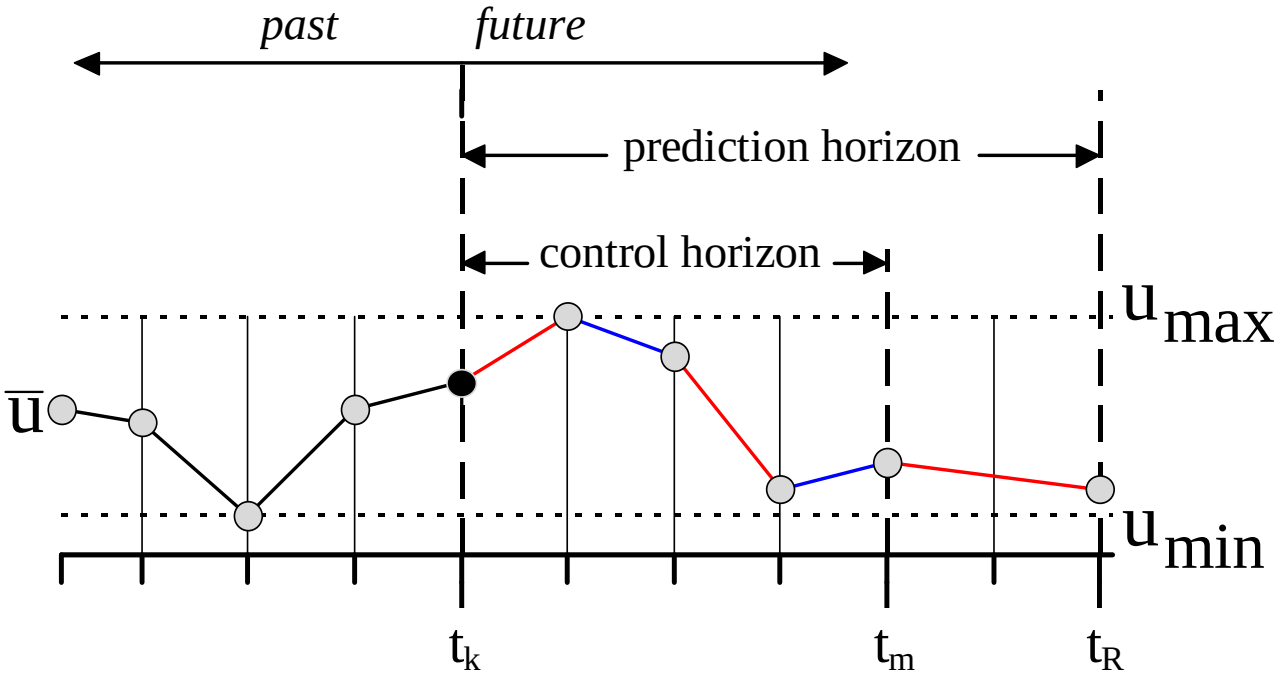
$$\mathbf{u}(i) = \mathbf{u}(k+m-1) \qquad \forall\ i > k+m-1$$

$$\dot{x}(t) = \left\| min\left\{0, \left(\mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} + \Delta\mathbf{T}_{min}^{t,*} \right)\right\}\right\| + \left\| min\left\{0, \left(\bar{\mathbf{T}}^{t} - \mathbf{T}^{t}(t) + \Delta\mathbf{T}_{max}^{t,*} \right)\right\}\right\| \qquad t \in [k,\ldots,k+R]$$

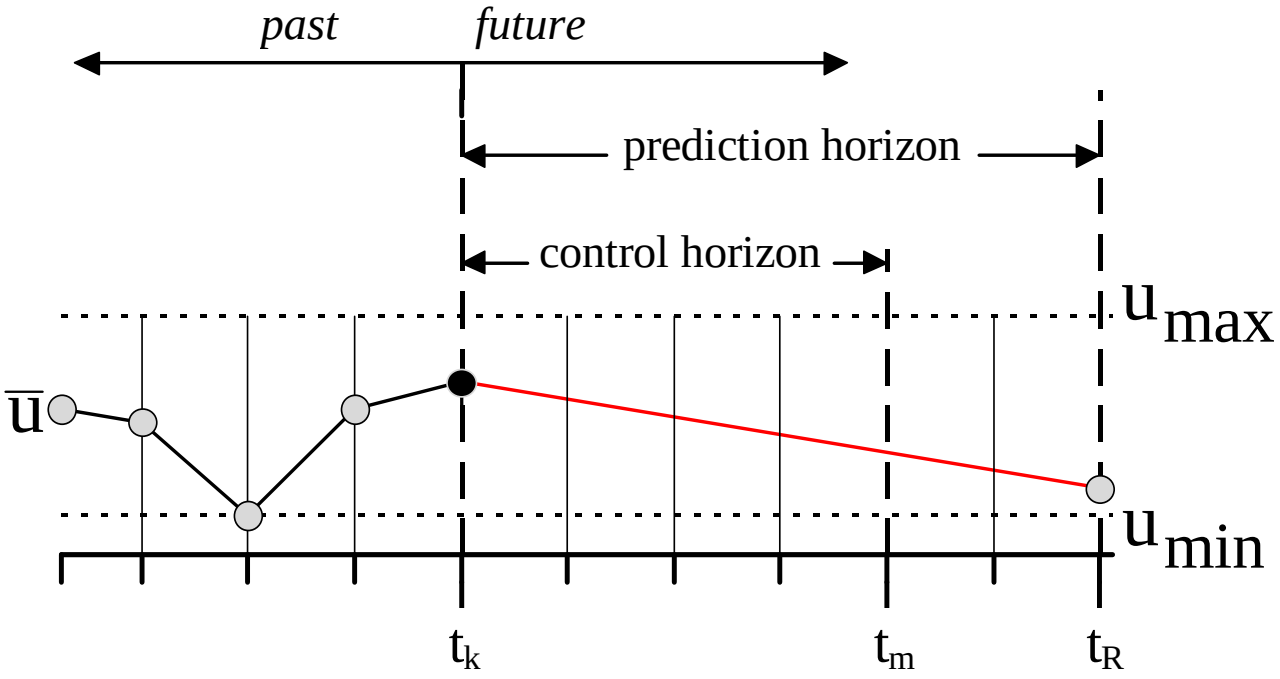
$$x(t)\big|_{k+R} = 0 \qquad t=k+R$$

NMPC with Piecewise-Continuous Control-Vector Parameterization

SPCCVLP: Local Parameterization (linear)



SPCCVGP: Global Parameterization (linear)



FORMULATION OF NMPC WITH Piecewise-Continuous Control-Vector Parameterization

$$\underset{\mathbf{u}(i),\ldots,\mathbf{u}(i+m-1)}{\textit{minimize}} \quad \Phi\left(\mathbf{u}(i)\right) = \int_{t_i}^{t_{k+R}} \left\| \mathbf{T}_k^{t,opt}(\infty) - \mathbf{T}_{MP}^{t}(i) \right\| dt \qquad i=k,\ldots,k+R$$

$$\mathbf{T}_{MP}^{t}(i) = \mathbf{T}_{M}^{t}(i) + \mathbf{d}(k) \qquad i=k,\ldots,k+R$$

$$\mathbf{d}(k) = \mathbf{T}_{P}^{t}(k) - \mathbf{T}_{M}^{t}(k)$$

$$\dot{\mathbf{x}}(t,z) = \boldsymbol{f}\left(\mathbf{x}(t,z), \partial\mathbf{x}(t,z)/\partial z, \mathbf{u}(t), \mathbf{p}, \mathbf{d}\right) \qquad t \in [k,\ldots,k+R]$$

$$\mathbf{x}(k,z) = \bar{\mathbf{x}}(z)$$

$$\Delta\mathbf{T}_{min}^{t,*} \le \left\{ \mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} \right\} \le \Delta\mathbf{T}_{max}^{t,*} \qquad t \in [k,\ldots,k+R]$$

$$(1-r)\bar{\mathbf{u}} \le \mathbf{u}(t) \le (1+r)\bar{\mathbf{u}} \qquad t \in [k,\ldots,k+R]$$

$$\dot{x}(t) = \left\| min\left\{0, \left(\mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} + \Delta\mathbf{T}_{min}^{t,*} \right)\right\}\right\| + \left\| min\left\{0, \left(\bar{\mathbf{T}}^{t} - \mathbf{T}^{t}(t) + \Delta\mathbf{T}_{max}^{t,*} \right)\right\}\right\| \qquad t \in [k,\ldots,k+R]$$

$$x(t)\big|_{k+R} = 0 \qquad t=k+R$$

$$\dot{y}(t) = \left\| max\left\{0, \left(\left| \mathbf{u}(t) - \bar{\mathbf{u}} \right| - r\bar{\mathbf{u}} \right)\right\}\right\| \qquad t \in [k,\ldots,k+R]$$

$$y(t)\big|_{k+R} = 0 \qquad t=k+R$$

for SPCCVLP:

$$\mathbf{u}(t) = \boldsymbol{P}_i(t) \qquad t \in [t_i,\ldots,t_{i+1}]$$

$$\boldsymbol{P}_i(t) = \boldsymbol{P}_{i-1}\left(t^{-}\right) \qquad t^{-} \in [t_{i-1},\ldots,t_i]$$

$$\frac{d\boldsymbol{P}_i(t)}{dt} = \frac{d\boldsymbol{P}_{i-1}\left(t^{-}\right)}{dt} \qquad t^{-} \in [t_{i-1},\ldots,t_i]$$

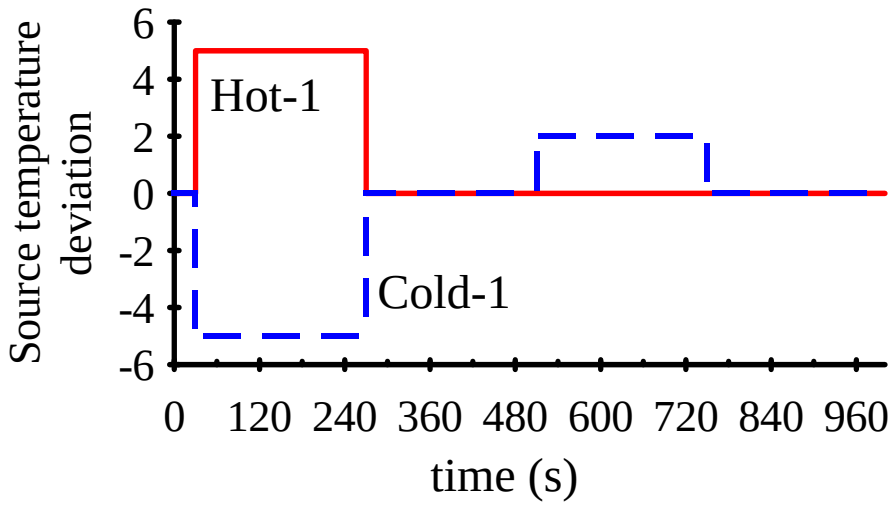
for SPCCVGP:

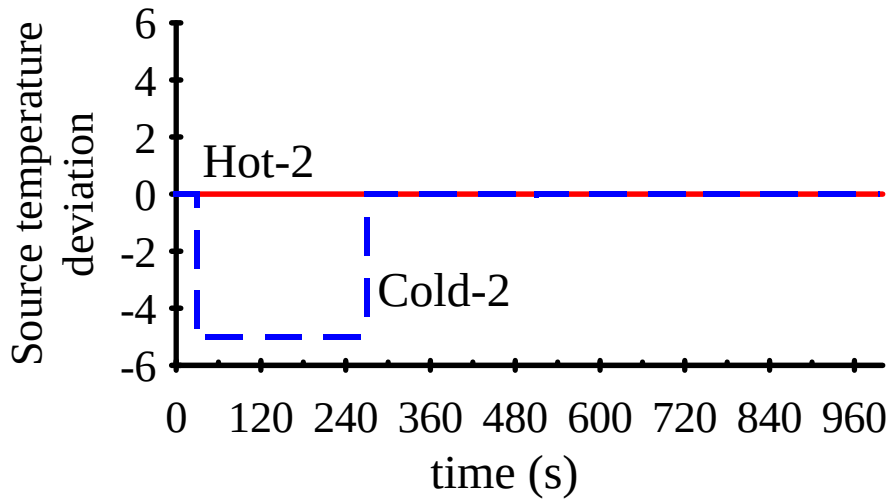
$$\mathbf{u}(t) = \boldsymbol{P}_k(t) \qquad t \in [t_k,\ldots,t_{k+R}]$$

$$\boldsymbol{P}_k(t) = \boldsymbol{P}_{k-1}\left(t^{-}\right) \qquad t^{-} \in [t_{k-1},\ldots,t_{k-1+R}]$$

$$\frac{d\boldsymbol{P}_k(t)}{dt} = \frac{d\boldsymbol{P}_{k-1}\left(t^{-}\right)}{dt} \qquad t^{-} \in [t_{k-1},\ldots,t_{k-1+R}]$$

Consecutive Disturbances Introduced in the Source-Stream Temperatures

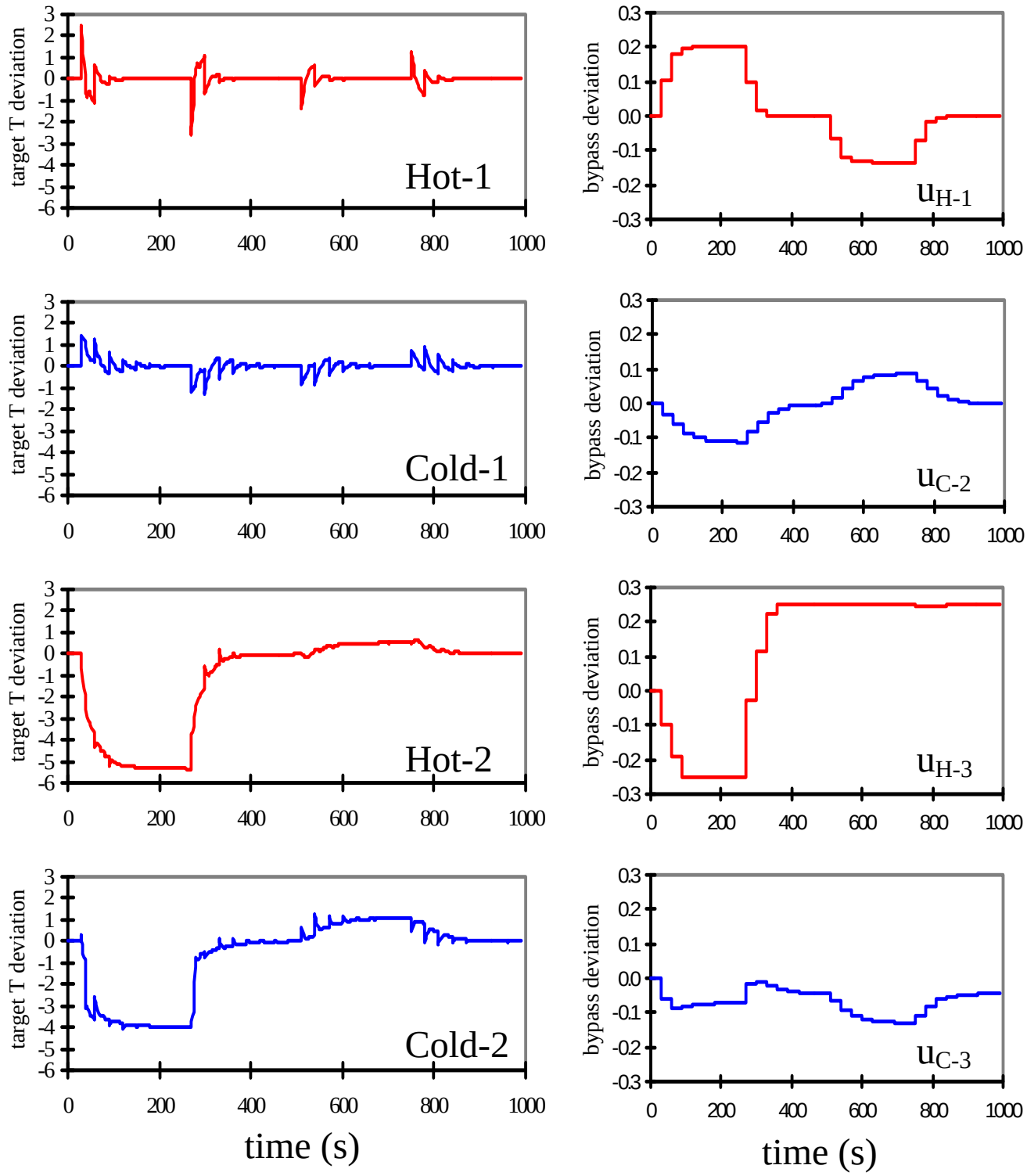




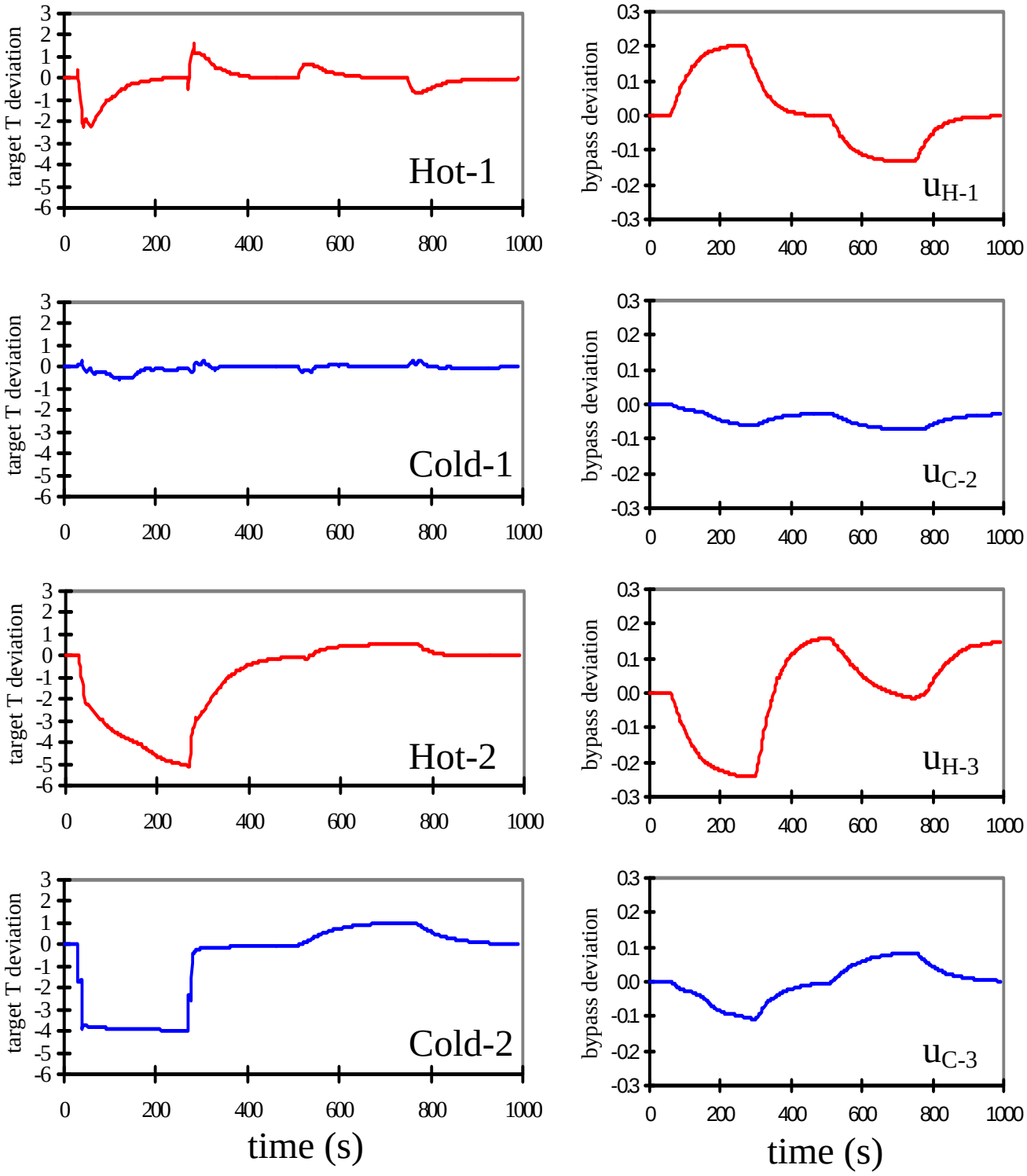
Some parameters used in NMPC schemes

$\Delta t = t_{k+1} - t_k = 30$ s	m = 3 steps	R = 4 steps
$t_R - t_m = \Delta t$	r = 0.25	r* = 0.1

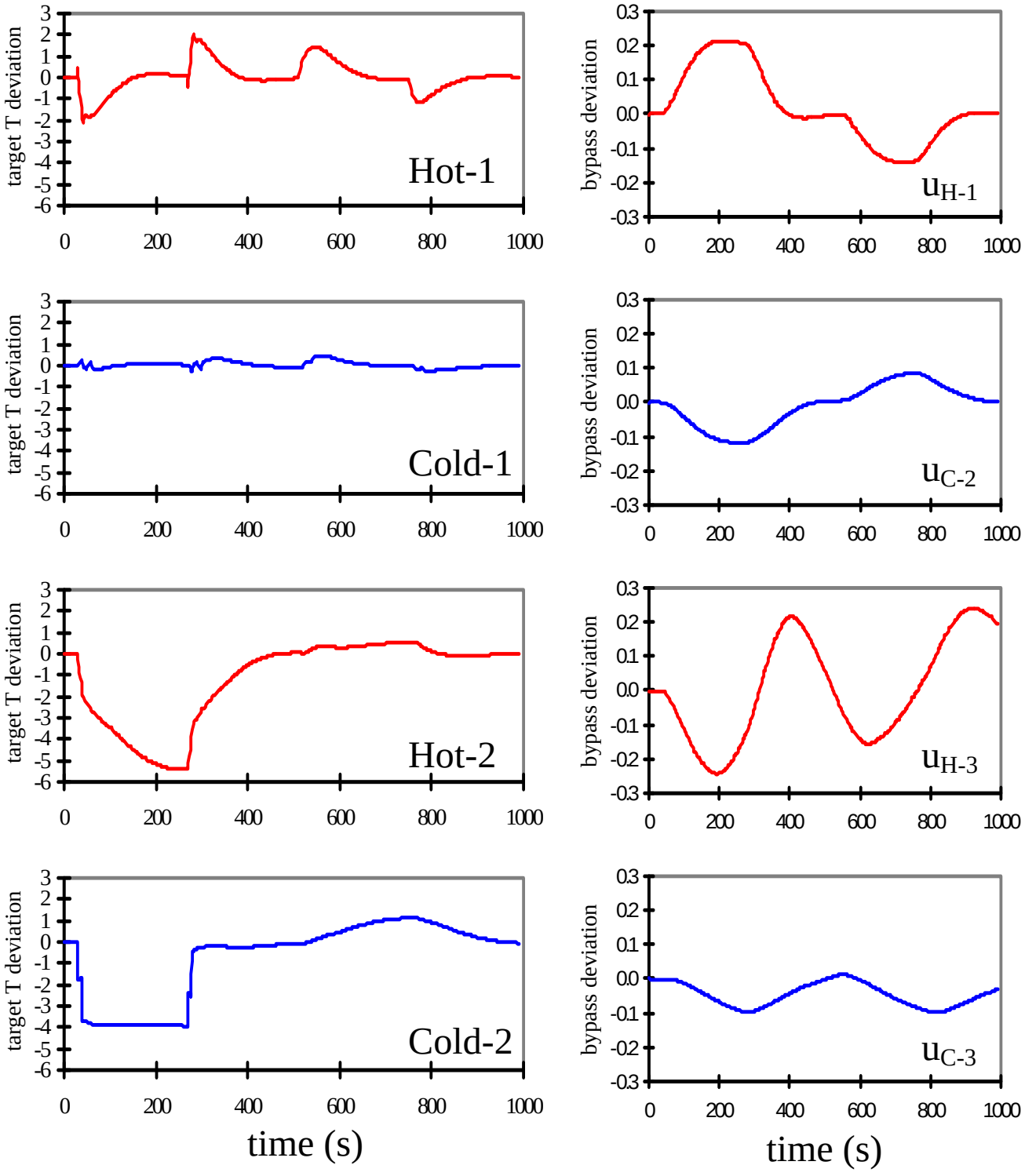
NMPC
without Plant/Model Mismatch



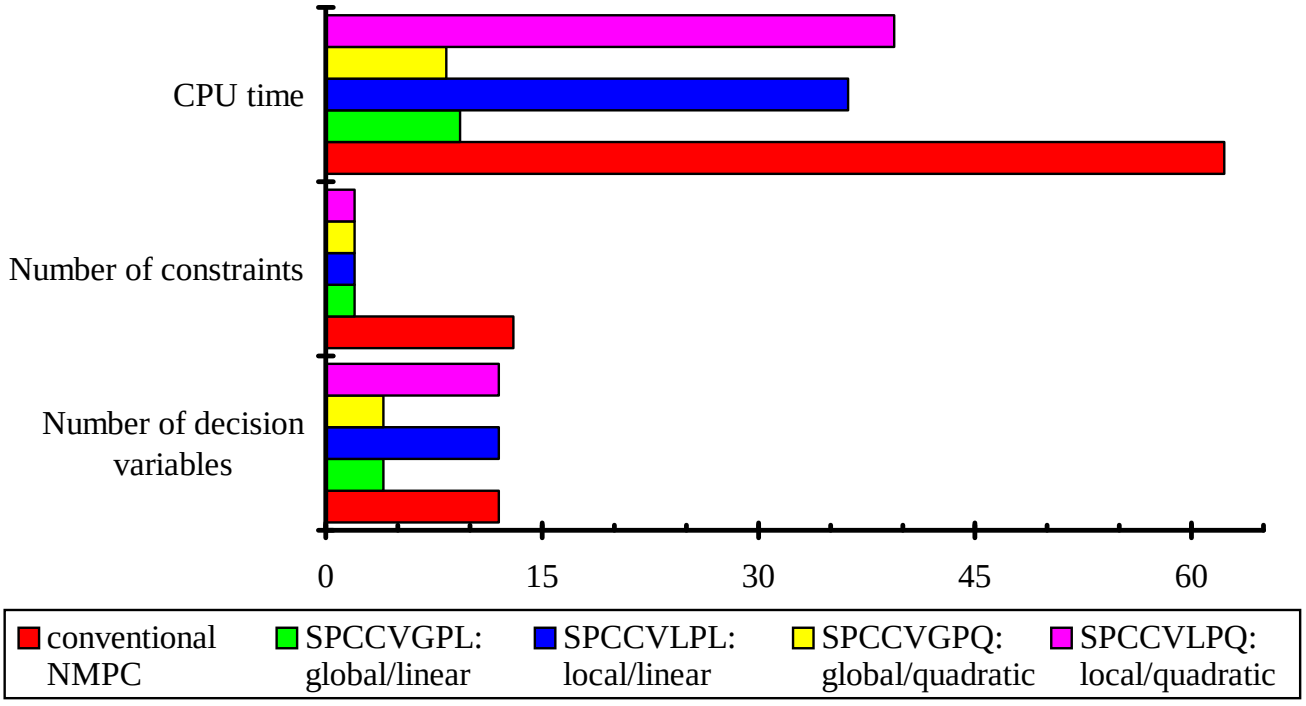
NMPC / SPCCVGP
global & linear parameterization
without Plant/Model Mismatch



NMPC / SPCCVGP
global & quadratic parameterization
without Plant/Model Mismatch



Comparison of NMPC with SPCCVP`s



ACKNOWLEDGMENT

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CONCLUSIONS

- Retrofit design of a HEN was performed considering placement of manipulated variables (bypasses) in order to increase HEN`s flexibility and resiliency.
- NMPC of the retrofit HEN modeled with PDEs was studied.
- A steady-state optimization algorithm, which determines feasible (optimal) post-disturbance target temperatures, was combined with the NMPC algorithm.
- The conventional NMPC scheme satisfied all the temporal and steady-state hard/soft target-temperature constraints even in the presence of a significant plant/model mismatch, however with nonsmooth response.
- A version of the classical NMPC scheme, NMPC/SPCCVGP, which has a piecewise-continuous linear-global parameterization of the control-vector, showed a superior performance with regard to smoothness of the response and significant reduction in computational load.
- Quadratic-global parameterization performed slightly better than the linear-global parameterization in terms of the smoothness of the response at no additional computational load.
- Local parameterizations (linear or quadratic) also gave smoother responses, however with no benefit with respect to computational load compared to the load in the conventional NMPC.